OSCILLATIONS

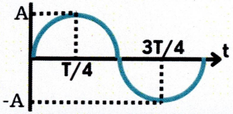
Comparison of Linear & Angular S.H.M

Linear S.H.M	Angular S.H.M
$F \propto -x$ $\quad F = -kx$ where k is restoring force constant	$\tau \propto -\theta$ $\quad \tau = -C\theta$ Where C is restoring torque constant
$a = -\frac{k}{m}x$	$\alpha = -\frac{C}{I}\theta$
$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$	$\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$
$a = -\omega^2 x$	$\alpha = -\omega^2 \theta$
$\omega^2 = \frac{k}{m}$ $\quad \omega = \frac{2\pi}{T} = 2\pi f = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{C}{I}} = 2\pi n = \frac{2\pi}{T}$

Angular Frequency

T=Time period f=frequency m=mass of particle K=force constant	$\omega = \frac{2\pi}{T} = 2\pi f = \sqrt{\frac{k}{m}}$

Displacement



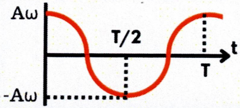
$y = A\sin(\omega t + \phi)$

A=amplitude

ϕ = initial phase

(at mean position $\phi = 0$)

Velocity

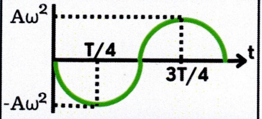


$y = A\sin(\omega t + \phi)$

$v = A\omega\sqrt{A^2 - y^2}$

(in terms of position)

Acceleration



$a = -A\omega^2 \sin(\omega t + \phi)$

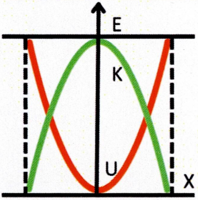
$a = -\omega^2 y$

Time (t)	0	T/4	2T/4	3T/4
Displacement	0(min)	A(max)	0(min)	A(max)
Velocity	Aω(max)	0(min)	-Aω(max)	0(min)
Acceleration	0(min)	$-A\omega^2$(max)	0(min)	$A\omega^2$(max)

Kinetic Energy	$K = \frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}k(a^2 - x^2)$

Potential Energy	$\frac{1}{2}kx^2$
Total Mechanical Energy	$\frac{1}{2}ka^2$(constant)

Energy diagram

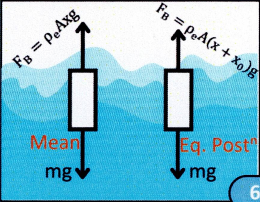


$U = \frac{1}{2}Kx^2$	
$K = \frac{1}{2}K(A^2 - X^2)$	
$E = \frac{1}{2}KA^2$	

S.H.M.

$K = \frac{1}{2}m\omega^2(x^2 + A^2\cos^2\omega t)$	$U = \frac{1}{2}m\omega^2(x^2 + A^2\sin^2\omega t)$

Time period for body in water



$$T = 2\pi\sqrt{\frac{\rho_s h}{\rho_l g}}$$

ρ_s = density of object in water

ρ_L = displacement from mean position

Simple Pendulum

$g_{eff} = g$, when whole system is at rest

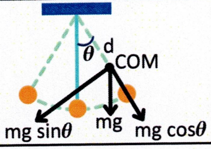
$g_{eff} = \sqrt{a^2 + g^2}$, in horizontal acceleration

$g_{eff} = g - a$, in deaccelerating refrence frame

$g_{eff} = g + a$, in accelerating reference frame

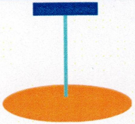
$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

Compound Pendulum



$$T = 2\pi\sqrt{\frac{l}{mgd}}$$

Torsional pendulum



$$T = 2\pi\sqrt{\frac{I}{C}}$$

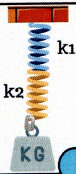
Simple pendulum of large length

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$$

Spring

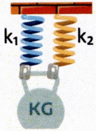
Series Combination

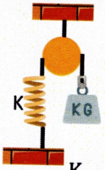
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$



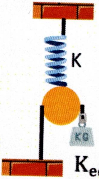
Parallel Combination

$$k = k_1 + k_2$$



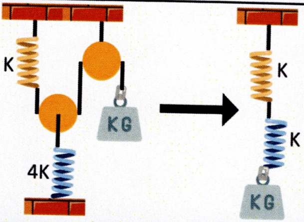


$K_{eq} = K$



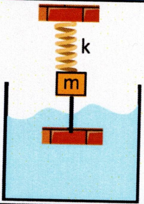
$K_{eq} = \frac{K}{4}$

$K_{eq} = 4K$



$$T = 2\pi\sqrt{\frac{2m}{K}}$$

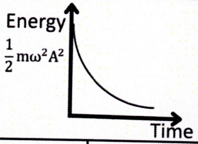
Damped Spring-mass System



For low speeds it is reasonable to write the frictional force as:

$F_f = -bv$ (b in kg/s)

Forced Oscillator



$$x_{(t)} = Ae^{-bt/2m}\cos(\omega' t + \phi)$$

where $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

Amplitude of Oscillation

$$Ae^{-bt/2m}$$

Total Energy

$$E = \frac{1}{2}KA^2e^{-bt/2m}$$

Forced Damped Oscillations

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = F_0\sin\omega_d t$$

$$A = \frac{F_0}{\sqrt{(\omega^2 - \omega_d^2)^2 + \left(\frac{b\omega_d}{m}\right)^2}}$$